III GEOMETRICAL OLYMPIAD IN HONOUR OF I.F.SHARYGIN
THE CORRESPONDENCE ROUND

1. (8) A triangle is cut into several (not less than two) triangles. One of them is isosceles (not equilateral), and all others are equilateral. Determine the angles of the original triangle.

2. (8) Each diagonal of a quadrangle divides it into two isosceles triangles. Is it true that the quadrangle is a diamond?

3. (8-9) Segments connecting an inner point of a convex non-equilateral n-gon to its vertices divide the n-gon into n equal triangles. What is the least possible n?

4. (8) Does a parallelogram exist such that all pairwise meets of bissectors of its angles are situated outside it?

5. A non-convex n-gon is cut into three parts by a straight line, and two parts are put together so that the resulting polygon is equal to the third part. Can n be equal to:

   a) (8) five?

   b) (8-10) four?

6. a) (8-9) What can be the number of symmetry axes of a checked polygon, that is, of a polygon whose sides lie on lines of a list of checked paper? (Indicate all possible values.)

   b) (10-11) What can be the number of symmetry axes of a checked polyhedron, that is, of a polyhedron consisting of equal cubes which border one to another by plane facets?

7. (8-9) A convex polygon is circumscribed around a circle. Points of contact of its sides with the circle form a polygon with the same set of angles (the order of angles may differ). Is it true that the polygon is regular?

8. (8-9) Three circles pass through a point P, and the second points of their intersection A, B, C lie on a straight line. Let A_1, B_1, C_1 be the second meets of lines AP, BP, CP with the corresponding circles. Let C_2 be the meet of lines AB_1 and BA_1. Let A_2, B_2 be defined similarly. Prove that the triangles $A_1B_1C_1$ and $A_2B_2C_2$ are equal.

9. (8-9) Suppose two convex quadrangles are such that the sides of each of them lie on the middle perpendiculars to the sides of the other one. Determine their angles.

10. (8-9) Find the locus of centers of regular triangles such that three given points A, B, C lie respectively on three lines containing sides of the triangle.

11. (8-10) A boy and his father are standing on a seashore. If the boy stands on his tiptoes, his eyes are at a height of 1 m above sea-level, and if he seats on father's shoulders, they are at a height of 2 m. What is the ratio of distances visible for him in two cases? (Find the answer to 0.1, assuming that the radius of Earth equals 6000 km.)

12. (9-10) A rectangle $ABCD$ and a point P are given. Lines passing through A and B and perpendicular to PC and PD respectively, meet at a point Q. Prove that $PQ \perp AB$.

13. (9-10) On the side AB of a triangle ABC, two points X, Y are chosen so that $AX = BY$. Lines CX and CY meet the circumcircle of the triangle, for the second time, at points U and V. Prove that all lines UV (for all X, Y, given A, B, C) have a common point.

14. (9-11) In a trapezium with bases AD and BC, let P and Q be the middles of diagonals AC and BD respectively. Prove that if $\angle DAQ = \angle CAB$ then $\angle PBA = \angle DBC$.

15. (9-11) In a triangle ABC, let AA', BB' and CC' be the bissectors. Suppose $A'B' \cap CC' = P$ and $A'C' \cap BB' = Q$. Prove that $\angle PAC = \angle QAB$.

16. (9-11) On two sides of an angle, points A, B are chosen. The middle M of the segment AB belongs to two lines such that one of them meets the sides of the angle at points A_1, B_1, and the other at points A_2, B_2. The lines A_1B_2 and A_2B_1 meet AB at points P and Q. Prove that M is the middle of PQ.

17. (9-11) What triangles can be cut into three triangles having equal radii of circumcircles?

18. (9-11) Determine the locus of vertices of triangles which have prescribed orthocenter and center of circumcircle.

19. (10-11) Into an angle A of size α, a circle is inscribed tangent to its sides at points B and C. A line tangent to this circle at a point M meets the segments AB and AC at points P and Q respectively. What is the minimum α such that the inequality $S_{PAQ} < S_{BMC}$ is possible?

20. (11) The base of a pyramid is a regular triangle having side of size 1. Two of three angles at the vertex of the pyramid are right. Find the maximum value of the volume of the pyramid.

21. (11) There are two pipes on the plane (the pipes are circular cylinders of equal size, 4 m around). Two of them are parallel and, being tangent one to another in the common generatrix, form a tunnel over the plane. The third pipe is perpendicular to two others and cuts out a chamber in the tunnel. Determine the area of the surface of this chamber.