# VIII Geometrical Olympiad in honour of I.F.Sharygin
# Final round. Second day. 8 form.

*Ratmino, 2012, August 1*

5. Do there exist a convex quadrilateral and a point $P$ inside it such that the sum of distances from $P$ to the vertices of the quadrilateral is greater than its perimeter?

6. Let $\omega$ be the circumcircle of triangle $ABC$. A point $B_1$ is chosen on the prolongation of side $AB$ beyond point $B$ so that $AB_1 = AC$. The angle bisector of $\angle BAC$ meets $\omega$ again at point $W$. Prove that the orthocenter of triangle $AWB_1$ lies on $\omega$.

7. The altitudes $AA_1$ and $CC_1$ of an acute-angled triangle $ABC$ meet at point $H$. Point $Q$ is the reflection of the midpoint of $AC$ in line $AA_1$; point $P$ is the midpoint of segment $A_1C_1$. Prove that $\angle QPH = 90°$.

8. A square is divided into several (greater than one) convex polygons with mutually different numbers of sides. Prove that one of these polygons is a triangle.

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# Final round. Second day. 9 form.

*Ratmino, 2012, August 1*

5. Let $ABC$ be an isosceles right-angled triangle. Point $D$ is chosen on the prolongation of the hypothenuse $AB$ beyond point $A$ so that $AB = 2AD$. Points $M$ and $N$ on side $AC$ satisfy the relation $AM = NC$. Point $K$ is chosen on the prolongation of $CB$ beyond point $B$ so that $CN = BK$. Determine the angle between lines $NK$ and $DM$.

6. Let $ABC$ be an isosceles triangle with $BC = a$ and $AB = AC = b$. Segment $AC$ is the base of an isosceles triangle $ADC$ with $AD = DC = a$ such that points $D$ and $B$ share the opposite sides of $AC$. Let $CM$ and $CN$ be the bisectors in triangles $ABC$ and $ADC$ respectively. Determine the circumradius of triangle $CMN$.

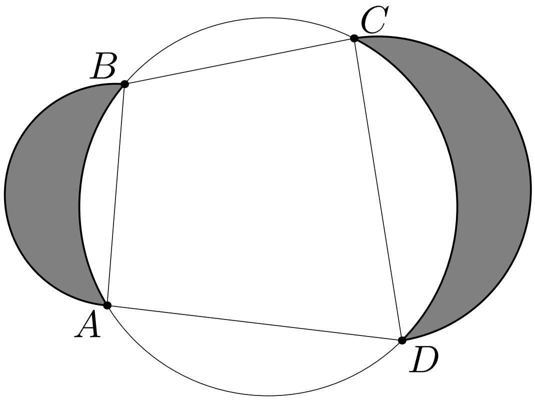
7. A convex pentagon $P$ is divided by all its diagonals into ten triangles and one smaller pentagon $P'$. Let $N$ be the sum of areas of five triangles adjacent to the sides of $P$ decreased by the area of $P'$. The same operations are performed with the pentagon $P'$; let $N'$ be the similar difference calculated for this pentagon. Prove that $N > N'$.

8. Let $AH$ be an altitude of an acute-angled triangle $ABC$. Points $K$ and $L$ are the projections of $H$ onto sides $AB$ and $AC$. The circumcircle of $ABC$ meets line $KL$ at points $P$ and $Q$, and meets line $AH$ at points $A$ and $T$. Prove that $H$ is the incenter of triangle $PQT$.

# VIII Geometrical Olympiad in honour of I.F.Sharygin
# Final round. Second day. 10 form.

*Ratmino, 2012, August 1*

5. A quadrilateral $ABCD$ with perpendicular diagonals is inscribed into a circle $\omega$. Two arcs $\alpha$ and $\beta$ with diameters $AB$ and $CD$ lie outside $\omega$. Consider two crescents formed by the circle $\omega$ and the arcs $\alpha$ and $\beta$ (see Figure). Prove that the maximal radii of the circles inscribed into these crescents are equal.




6. Consider a tetrahedron $ABCD$. A point $X$ is chosen outside the tetrahedron so that segment $XD$ intersects face $ABC$ in its interior point. Let $A'$, $B'$, and $C'$ be the projections of $D$ onto the planes $XBC$, $XCA$, and $XAB$ respectively. Prove that $A'B' + B'C' + C'A' \leq DA + DB + DC$.

7. Consider a triangle $ABC$. The tangent line to its circumcircle at point $C$ meets line $AB$ at point $D$. The tangent lines to the circumcircle of triangle $ACD$ at points $A$ and $C$ meet at point $K$. Prove that line $DK$ bisects segment $BC$.

8. A point $M$ lies on the side $BC$ of square $ABCD$. Let $X$, $Y$, and $Z$ be the incenters of triangles $ABM$, $CMD$, and $AMD$ respectively. Let $H_x$, $H_y$, and $H_z$ be the orthocenters of triangles $AXB$, $CYD$, and $AZD$. Prove that $H_x$, $H_y$, and $H_z$ are collinear.