Right or Wrong: Prime Factorization

1. Given integers *a*, *b*, *c* such that $a=b^2$ and $a=c^3$. Does this imply there exist an integer *d* such that $a=d^6$?

2. Let *a, b, c* positive integers such that GCD of any two of them is greater then 1. Can GCD(*a, b, c*)=1?

3. Can the product of two consecutive positive integers be divisible by 1000 if both integers are less then 1000?

4 a) Given three positive integers such that none of them is a divisor of another. Can each of them be a divisor of the two others product?

b) Given ten positive integers such that none of them is a divisor of another. Can each of them be a divisor of the product of any two others?

5. a) Does there exist a strictly increasing sequence of integers $n_1, n_2, \ldots, n_9$ such that the sequence $GCD(n_1, n_2), GCD(n_2, n_3), \ldots, GCD(n_8, n_9)$ is strictly decreasing?

b) Does there exist a strictly increasing sequence of integers $n_1, n_2, \ldots, n_9$ such that the sequence $LCM(n_1, n_2), LCM(n_2, n_3), \ldots, LCM(n_8, n_9)$ is strictly decreasing?

6. a) Can one place 8 integers at the cube vertices such that (one of the integers be a divisor of an other <=> both integers be on the same edge).

b) Can one place 9 integers at the 9-gon vertices such that (one of the integers be a divisor of an other <=> both integers be on the same side).

7. Remove one of 100 factors from the product $1!\cdot 2!\cdot \ldots \cdot 100!$ to get a perfect square.

8. Integers from 2 to 10001 are stored in 10000 cells. One knows GCD for any two cells. Is it enough to find out the number in each cell?

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