Functional Equations

Definition. Let $\mathbb{R}_{pos}$ be the set of all positive real numbers.

Substituting numbers

1. The function $f:\mathbb{R}\to\mathbb{R}$ satisfies $f(x+y)=f(x)+f(y)+80xy \quad \forall x,y\in\mathbb{R}$.
a) Find *f*(0,5), if *f*(0,25) = 2,5.
b) Find *f*(6), if *f*(1,5)=90.

2. The function $f:\mathbb{R}\to\mathbb{R}$ satisfies $f(3x+1)=f(x)+3x+1{,}5 \quad \forall x\in\mathbb{R}$. It is known that *f*(40)=67,5. Find **a)** *f*(13), **b)** *f*(1).

3. The function $f:\mathbb{R}\to\mathbb{R}$ satisfies $f(xy)=f(x)f(y)-f(x)-y^2+1 \quad \forall x,y\in\mathbb{R}$. Find *f*(0).

4. Does there exist a function $f:\mathbb{R}_{pos}\to\mathbb{R}$, such that $f(xy)=4f(x)-3f(y)+2$ for every positive numbers *x* and *y* ?

5. The function $f:\mathbb{R}_{pos}\to\mathbb{R}$ satisfies $f(xy)=f(x)+f(y)$ for every positive numbers *x* and *y*. Find *f*(20), if *f*(0,05) = 10.

6. The function $f:\mathbb{R}_{pos}\to\mathbb{R}_{pos}$ is such that *f*(1) +*f*(2) = 10 and $f(x+y)=f(x)+f(y)+2\sqrt{f(x)f(y)} \quad \forall x,y\in\mathbb{R}_{pos}$. Find $f(2^{1000})$.

7. The function $f:\mathbb{R}\to\mathbb{R}$ is defined $\forall x\neq 1$ and $(x-1)f\left(\frac{x+1}{x-1}\right)=x+f(x)$. Find *f*(–1).

Substituting expressions

8. Find all functions $f:\mathbb{R}\to\mathbb{R}$ such that $f(x+y)+f(x-y)=x^2+y^2 \quad \forall x,y\in\mathbb{R}$.

9. Find all functions $f:\mathbb{R}\to\mathbb{R}$ such that $f(2x+1)=4x^2+14x+7 \quad \forall x\in\mathbb{R}$

10. Find all functions $f:\mathbb{R}\to\mathbb{R}$ such that $f(x)=x^2+2f(1-x) \quad \forall x\in\mathbb{R}$

11. The function $f:\mathbb{R}\to\mathbb{R}$ is defined $\forall x\neq 0$ and satisfies $f(x)+f\left(\frac{x-1}{x}\right)=2x \quad \forall x\in\mathbb{R}, x\neq 0$. Find *f*(*x*).

Auxiliary function

12. Find all functions $f:\mathbb{R}_{pos}\to\mathbb{R}$ such that $f(xy)+f\left(\frac{x}{y}\right)=(\ln(x))^2+(\ln(y))^2 \quad \forall x,y\in\mathbb{R}_{pos}$

Extra problems

RNO1998. Find every $a\in\mathbb{R}$ such that there exists a non-constant function $f:\mathbb{R}\to\mathbb{R}$ satisfying $f(a(x+y))=f(x)+f(y)$.

RNO2014. Given the function $f:\mathbb{R}\to\mathbb{R}$ such that $(f(x))^2\leq f(y) \quad \forall x>y$, prove that $0\leq f(x)\leq 1 \quad \forall x\in\mathbb{R}$

FE1. Find all functions $f:\mathbb{R}\to\mathbb{R}$ such that $f(x+y)+f(x-y)=2f(x)\cos(y) \quad \forall x,y\in\mathbb{R}$