Test 1

1. For the sequence $a_1, a_2, \ldots, a_{2015}$ holds $a_{i+1}+a_{i-1}=2a_i$ for each $i=2,3, \ldots, 2014$. It is known that $a_1=1$, $a_{2015}=1000$. Find a_{1000}.

2. Two function graphs $y=x^3+ax^2+bx+c$ and $y=x^3+ex^2+fx+g$ have a finite number of common points. Find the maximal number of these common points.

3. A matrix M of size 3×3 has two of its eigenvalues 2 and 5, det(M)=100. Find tr(M).

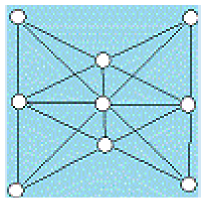
4. A function $f(x)$ has 10 roots, its derivative $f'(x)$ is continuous and has no common roots with $f(x)$. What minimal number of local maximums $f(x)$ can have?

5. z and w are two complex numbers, $|z|=2$, $|w|=5$. Find the minimal possible value for $|z^2+w^2|$.

6. Given 3 real numbers. If one increase each of original numbers by 1, their product increases also by 1. If one increase each of original numbers by 2, their product increases also by 2. In fact each of original numbers was increased by 3. Find the increment of the product.

7. On each of 100 cards is written a real number. For each pair of cards John calculates two results: the product of numbers on the cards and the sum of numbers on the cards. John writes down the product onto the board #1 and the sum onto the board #2. At last he has as many numbers on each board as there are card pairs. (Some of numbers can coincide). Among the numbers on the board #1 there are exactly 100 negative numbers, and among the numbers on the board #2 there are exactly 100 negative numbers. How many positive numbers are among the numbers on the board #1? (If a product or a sum is zero it is neither negative nor positive). .

8. 9 apples lie on the table making 10 straight rows with 3 apples in each (see the picture). It is known that 9 of the rows are of the same total weight while the 10th row has a different weight. An electronic scales are available. By paying a dollar one can find the weight of any group of apples. What minimal amount one can pay to find out which of rows has a different weight? (One should point out the row, not find the weight of the row).



9. We call a nine-digit number *good* if it has a digit that can be moved to *another* place so that the new nine-digit number has its digits in a strictly ascending order. How many good numbers are there in total?

10. A positive integer N can be transformed to the exponent by moving the last digit up (e.g., number 179 is transformed to 17^9, or 2007 transformed to 200^7). Find the maximal N for which the transformed number is divisible by N.

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